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Interactions between chromatic- and luminance-contrast-sensitive stereopsis mechanisms

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Abstract

It is well known that chromatic information can assist in solving the stereo correspondence problem. It has also been suggested that there are two independent first-order stereopsis mechanisms, one sensitive to chromatic contrast and the other sensitive to luminance contrast (*Vision Research 37* (1997) 1271). Could the effect of chromatic information on stereo correspondence be subserved by interactions between these mechanisms? To address this question, disparity thresholds (1/stereoacuity) were measured using 0.5 cpd Gabor patches. The stimuli possessed different relative amounts of chromatic and luminance contrast which could be correlated or anti-correlated between the eyes. Stereoscopic performance with these compound stimuli was compared to that with purely isoluminant and isochromatic stimuli at different contrasts. It was found that anti-correlated chromatic contrast severely disrupted stereopsis with achromatic stimuli and that anti-correlated luminance contrast severely disrupted stereopsis with chromatic stimuli. Less dramatic, but still significant, was the improvement in stereoacuity obtained using correlated colour and luminance contrast. These data are consistent with there being positive and negative interactions between chromatic and achromatic stereopsis mechanisms that take place after the initial encoding of disparity information, but before the extraction of stereoscopic depth. These interactions can be modelled satisfactorily assuming probability summation of depth sign information between independent mechanisms.

Keywords: Stereopsis; Colour; Depth perception; Isoluminance; Binocular vision

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1. Introduction

It has been known for some time that chromatic information can assist in solving the stereo correspondence problem (Treisman, 1962; Julesz, 1971; Ramachandran, Rao, Sriram, & Vidyasayar, 1973; Jordan, Geisler, & Bovik, 1990; Jordan & Bovik, 1991, 1992; Kovács & Julesz, 1992). The consensus of these studies is that the visual system favours stereoscopic matches that have similar chromaticities. One potential mechanism for this chromatic matching process is that visual features are somehow given a "label" based on their chromaticity and that matches which have the same label are favoured over those that do not.

An alternative mechanism is suggested by Simmons and Kingdom (1997). They provided evidence for two independent stereopsis mechanisms, one sensitive to (red–green) chromatic contrast, and the other sensitive to luminance contrast. Each of these mechanisms could produce an independent estimate of stereoscopic depth which is subsequently combined into a unified percept. Which of these two alternatives is more consistent with psychophysical data?

Most empirical studies of the relationship between colour vision and stereopsis have concentrated on either the nature of stereopsis at isoluminance or whether chromatic information can assist in solving the correspondence problem. A series of studies on the first of these themes has attempted to establish the existence of a chromatic stereopsis mechanism and to characterize its properties (Simmons & Kingdom, 1994, 1995, 1997; Kingdom & Simmons, 1996; Kingdom, Simmons, & Rainville, 1999; for a review see Kingdom & Simmons, 2000). The conclusion from these studies is that there exists a rudimentary chromatic stereopsis mechanism which is less contrast sensitive, has a more limited disparity range, poorer stereoacuity and poorer ability to

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encode stereoscopically defined shape than its luminance counterpart.

The most informative experiments on the second theme, namely whether chromatic information can assist in solving the correspondence problem, are those of Jordan, Bovik, and co-workers (Jordan et al., 1990; Jordan & Bovik, 1991, 1992). They established that similarity of chromaticity could be used to extend the range of perceived stereoscopic depths in ambiguous "wallpaper" stereograms (Jordan et al., 1990). They went on to demonstrate the theoretical usefulness of chromatic information in stereoscopic matching in computational investigations (Jordan & Bovik, 1991, 1992). Kovács and Julesz (1992) demonstrated that chromatic matching processes were in some cases powerful enough to reverse the effects of luminance anti-correlation in their so-called "meta-isoluminant" random-dot stereograms.

How can the results of experiments on stereopsis at isoluminance and experiments on the usefulness of chromatic information for solving the correspondence problem be unified? Could the enhancements of stereopsis found by using correlated colour information reflect interactions between the putative chromatic- and luminance-contrast sensitive stereopsis mechanisms as described by Simmons and Kingdom (1997), which initially process stereoscopic information separately?

To investigate this issue, we measured the effects of superimposed chromatic information on the precision of stereoscopic depth judgements. Performance was measured under a variety of conditions with isoluminant red–green stimuli, isochromatic yellow-black stimuli and compounds of the two. It was found that anti-correlations in colour contrast degraded luminance-based stereopsis and vice versa. It was also found that correlations between colour and luminance contrast improved stereoscopic performance. It is argued that these results can be explained (and modelled satisfactorily) in terms of interactions between chromatic- and luminance-contrast-sensitive stereopsis mechanisms and that there is no need to invoke a chromatic feature matching mechanism to explain the data.

2. Methods

2.1. Subjects

Subjects were the two authors. One (FK) is emmetropic and the other (DS) wore his prescribed optical correction. Both subjects are colour normal and highly experienced in stereoscopic depth discriminations.

2.2. Stimulus generation

The stimuli were generated using the VSG2/3F videographics card (Cambridge Research Systems) hosted by a Gateway 2000 P5 computer, and displayed on a BARCO Calibrator monitor.

2.3. Display calibration and contrast resolution

The VSG2/3F can display images with 8-bit-per-RGB-gun (256 level) resolution, the 8 bits being selected from 12-bit (4096 levels) linearized colour look-up-tables (CLUTs). Each gun on the monitor was calibrated using the Optical system (Cambridge Research Systems), which generated the 12-bit gamma-corrected CLUTs. The 12-bit CLUTs provided a contrast resolution of about 0.05% which is sufficient for measuring contrast thresholds. Whatever the contrast of the stimulus, it was always displayed with the full 8-bits, the intensities of which were suitably selected from the 12-bit CLUTs. Finally the VSG has a special facility whereby two stimuli, each defined by separate 8 bit CLUTS, can be displayed at the same time on different parts of the monitor screen. This feature was used to define separately the CLUTS of the two stereo-half images, which were displayed on either side of the vertical midline of the monitor and which in many conditions differed in their luminance/colour contrast.

2.4. Stimuli

The stimulus used was a Gabor patch, consisting of a vertically oriented sinusoidal carrier in luminance and/or colour modulated by a two-dimensional isotropic Gaussian envelope. The spatial frequency of the carrier was 0.5 cpd and the standard deviation of the envelope was 1°, resulting in a spatial bandwidth of approximately 1.1 octaves (full width at half maximum). The spatial parameters of the stimulus were designed to minimize luminance artifacts due to chromatic aberration (Scharff & Geisler, 1992). The carrier was always in sine phase relative to the envelope. The stimulus appeared in a high-contrast black [1] fixation circle of radius 3° which was present throughout the experiment.

Luminance contrast was generated by modulating the red and green guns of the monitor in spatial phase, whereas chromatic contrast was generated by modulating these guns in spatial anti-phase. Compound stimuli, i.e. stimuli with both luminance and chromatic contrast, were generated by specifying the luminance and chromatic contrasts separately and then calculating the appropriate gun modulations (see Simmons & Kingdom, 1997, for a more detailed description). The absolute phases of the luminance and chromatic modulations for all stimuli were separately randomized for each stimulus presentation. Thus if a compound Gabor patch was presented it would appear randomly with either 'bright

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[1] That is, zero output on all monitor guns.

red/dark green' bars, or 'bright green/dark red' bars, and with a random sign of contrast. In the conditions in which the two stereo-halves had different contrast polarities, the eye to which a given contrast polarity was presented was also randomized.

The luminance and chromatic contrasts reported are the Michelson contrasts (i.e. $(L_{\max} - L_{\min})/(L_{\max} + L_{\min})$) of the Gabor carrier before multiplication by the Gaussian envelope. The ratio of red to overall mean luminance, the $R/(R+G)$ ratio, was determined by the isoluminance setting (see below). Variations in $R/(R+G)$ ratio from low to high values resulted in the colour of the background field varying from greenish through yellow to reddish. The mean luminance of the background field and stimuli at the eye was 8.0 cd/m^2.

2.5. Stereo presentation

The two stereo-halves were presented on either side of the monitor screen separated by 11 cm. They were combined optically by a custom-built 8-mirror modified Wheatstone stereoscope similar to that used in a number of previous studies (e.g. Hess, Hayes, & Kingdom, 1997; Kingdom et al., 1999). All mirrors were cemented into position except for the two front mirrors whose position along the line of sight of the subject could be adjusted until fusion was accomplished. Viewing distance, as measured by the length of the path of light from the monitor screen to the eyes, was 55 cm.

2.6. Procedure

Stereoacuity was measured using a 2IFC (two interval forced-choice) procedure. Before each session the subject was required to adapt to a blank yellow screen at the appropriate $R/(R+G)$ ratio for one minute. In each trial one of the intervals contained a stimulus with $-1/2$ the stimulus disparity (behind fixation), while the other contained a stimulus with $+1/2$ the stimulus disparity (in front of fixation). The subject was required to indicate the interval in which the stimulus appeared in front. A tone accompanied each stimulus presentation to help define the presence of the stimuli, which were sometimes near detection threshold. A different tone indicated an incorrect response. Stimulus exposure duration was 500 ms. Stimulus onset and offset were abrupt. A standard 'two-up, one-down' staircase procedure was employed (Levitt, 1971) to obtain the disparity threshold. This procedure gives the threshold for the 70.7% correct performance level. The staircase was terminated after 12 reversals and the threshold calculated as the geometric mean disparity over the previous 10 reversals. At least three thresholds were measured for each condition, and unless stated otherwise the data points shown in the figures give the geometric means and geometric standard errors of these measurements.

3. Results

The first step was to determine the isoluminant point for this stereoacuity task for each subject. The method used was the "method of worst performance" (see Kingdom & Simmons, 1996). The advantage of this method over more photometrically based methods, such as heterochromatic flicker photometry, is that the spatial and temporal properties of the stimulus are matched in both the task of interest and the isoluminance determination. Stereoacuity was measured using a 25% colour contrast stimulus presented at a range of $R/(R+G)$ levels. The stereoacuity data were then fit with a smooth Gaussian and the position of the maximum of this Gaussian was taken as the $R/(R+G)$ value of the isoluminant point. Data from the two subjects are presented in Fig. 1. The isoluminant points for each subject were 0.46 and 0.51 for FK and DS respectively.

Having established the isoluminant point for each subject, the next step was to determine the dependence of stereoacuity on colour and luminance contrast for each subject. This experiment provided baseline data for comparison with subsequent experiments. To facilitate comparison, these data were fit with a smooth function of the following form:

$$D = \frac{\alpha}{(c-\beta)^{\gamma}} + \delta \tag{1}$$

where D is disparity threshold in arcmin, c is contrast as a percentage, and α, β, γ, δ are free parameters. Note that this function is equivalent to the conventional power-law relationship between disparity threshold and contrast (Legge & Gu, 1989), but with a simple translation of the axes to take into account asymptotic performance at very low and very high contrasts. As such it is essentially a smooth equivalent of the piece-wise straight line fits on log–log coordinates used by Kingdom and Simmons (1996) to describe similar stereoacuity data. This can be appreciated if logarithms of both sides of (1) are taken:

$$\log(D-\delta) = -\gamma\log(c-\beta) + \log\alpha \tag{2}$$

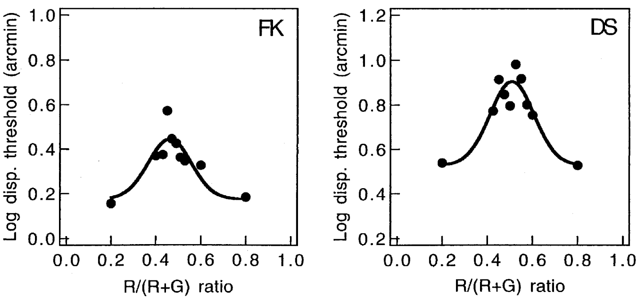

Fig. 1. Log disparity threshold in arcmin vs. $R/(R+G)$ ratio for both subjects. Stimulus colour contrast was 25%. The smooth curve is the best-fitting Gaussian to the data. Notice that the disparity ranges on the ordinate are slightly different for the two subjects.

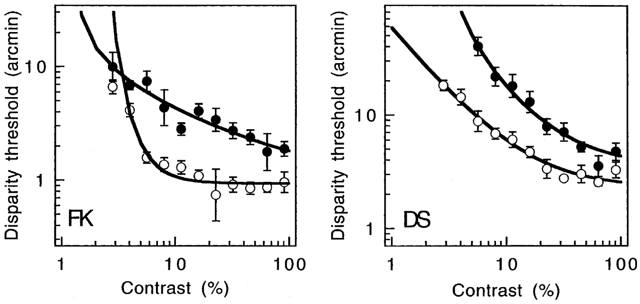

Fig. 2. Disparity threshold in arcmin vs. chromatic (filled circles) and luminance (unfilled circles) contrast in % for the two subjects. Error bars are standard errors on the geometric mean. Smooth curves are the best-fitting versions of Eq. (1) (see text) to each data set. The parameters of these fits are given in Table 1. Again notice that the disparity ranges on the ordinate are different for the two subjects.

This form of the relationship makes clearer the meaning of each parameter. β and δ are the positions of the asymptotes on the contrast and disparity threshold axes respectively, γ is the slope of the function on log–log axes and α is the intercept. The data are shown for two subjects in Fig. 2. In Table 1 the parameters of each fit are given. Note that, unlike in Kingdom and Simmons (1996), the data are not normalized to detection threshold. Consequently, performance is not strictly comparable between chromatic and achromatic performance at the same contrast level. This transformation would simply shift the chromatic and achromatic data relative to one another along the log-contrast axis.

Fig. 3 illustrates the effects of correlated and anti-correlated chromatic contrast on stereoacuity with a luminance Gabor. The luminance contrast of the stimulus was fixed at 8% for FK and 9.9% for DS. These intermediate contrast levels were chosen to allow room for both improvement and degradation in performance. There was no consistent improvement in stereoacuity with increasing amounts of added correlated chromatic contrast. The maximum improvements were factors of 1.28 and 1.27 for FK and DS respectively. Much more dramatic, however, was the degradation in stereoacuity caused by adding anti-correlated chromatic contrast to the luminance-defined stimulus. The maximum degradation was a factor of 12.26 for FK and 5.32 for DS.

Table 1
Parameters of the best-fitting versions of Eq. (1) to the data shown in Fig. 2

Subject/ condition	Best-fitting function parameters			
	α	β	γ	δ
FK lum	9.26	2.3	2.0	0.93
FK col	11.41	1.29	0.55	0.91
DS lum	53.06	0.05	1.15	2.28
DS col	137.8	2.36	1.10	3.41

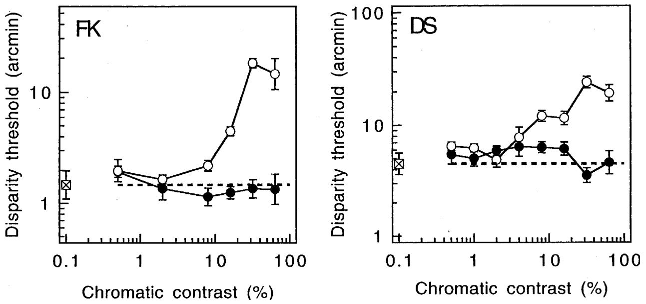

Fig. 3. Disparity threshold in arcmin vs. chromatic contrast as a percentage for a stimulus with fixed luminance contrast of 8% and 9.9% for FK and DS respectively. Filled circles represent correlated and unfilled circles anti-correlated chromatic contrast. Error bars as in previous figures. The baseline disparity threshold (luminance contrast only) is shown as the square and cross. The dashed horizontal line is the baseline level. Again the disparity ranges on the ordinate are different for the two subjects.

Having established that anti-correlated isoluminant colour contrast could disrupt stereopsis with a luminance-defined stimulus, it was necessary to establish if the obverse was true. That is, could colour-based stereopsis be disrupted by adding anti-correlated luminance information? The results are shown in Fig. 4. The fixed chromatic contrasts were 45% for FK and 33% for DS.

The dependence of stereoacuity for a high-contrast chromatic stimulus on the amount of added correlated and anti-correlated luminance contrast for FK was similar to that found using a fixed luminance contrast Gabor with correlated and anti-correlated chromatic contrast. There was a slight improvement with increasing correlated luminance contrast (a maximum improvement of a factor of 1.61) but a gradual disruption with increasing anti-correlated luminance contrast (a highly significant factor of 14.4 at maximum). Although the second subject (DS) did not perform a full experiment with fixed colour contrast the trends in his data were the same. It was found that when colour contrast was fixed at 33%, correlated luminance contrast of 9.9% improved stereoacuity by a factor of 1.66. Anti-

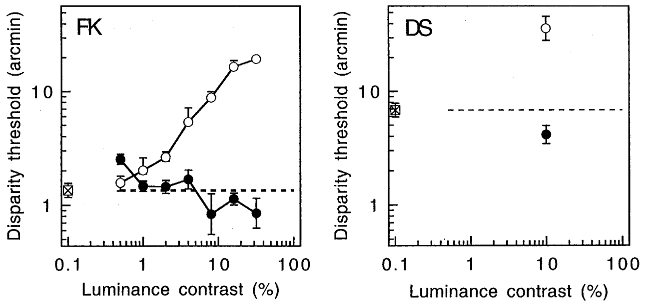

Fig. 4. As Fig. 3, except abscissa is luminance contrast (correlated or anti-correlated) for fixed chromatic contrast (45% and 33% for FK and DS respectively).

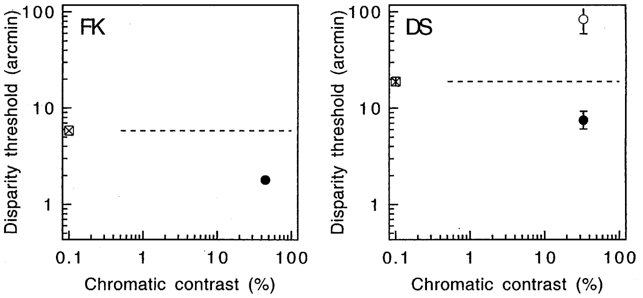

Fig. 5. Again as Fig. 3, except that the fixed luminance contrast (correlated or anti-correlated) is low (3%). FK could not obtain a threshold for the anti-correlated condition.

correlated luminance contrast of the same magnitude produced a factor of 4.10 degradation in stereoacuity.

Having found evidence that there was some evidence of improvement with correlated luminance contrast superimposed on fixed colour contrast we decided to return to the fixed luminance contrast data but this time use a much lower baseline contrast of 3%. The results are shown in Fig. 5.

Both subjects showed significant improvements in performance with added correlated colour contrast. These improvements were factors of 3.24 and 2.54 for FK and DS respectively.

This result prompted a further investigation on subject FK. Three conditions were compared. In the first stereoacuity was measured for luminance contrast stimuli as a function of contrast (as previously in Fig. 2). In the second a fixed isoluminant colour contrast of 15% was added to the luminance contrast stimuli. In the third the fixed colour contrast was 45%. The results are shown in Fig. 6.

It can be seen that, when the luminance contrast is low, the added colour contrast improves performance approximately to the levels expected if the colour contrast was presented alone. With the lower fixed colour contrast performance began to improve once the disparity threshold obtainable with the luminance contrast component alone was lower than that obtainable with the colour contrast component alone. This effect was not observed with the higher fixed colour contrast although, at the highest contrast of the luminance component, performance was not significantly different between the two conditions with colour contrast present and absent.

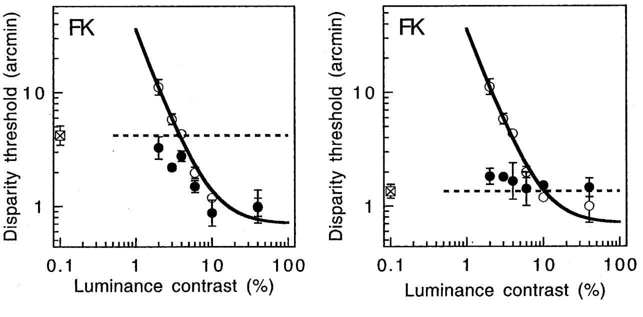

Fig. 6. Disparity threshold in arcmin vs. luminance contrast as a percentage for a stimulus containing either luminance contrast alone (unfilled circles) or a fixed colour contrast of 15% (left panel) or 45% (right panel) and a variable luminance contrast component (filled circles). Error bars as in previous figures. The baseline disparity threshold for 15% or 45% colour contrast only is shown as the square and cross. The dashed horizontal line is this same baseline level. The smooth solid line is the best fit of Eq. (1) to the luminance contrast only data (parameters: $\alpha = 30.94$; $\beta = 0.08$; $\gamma = 1.66$; $\delta = 0.70$).

4. Discussion

4.1. Comparison with previous studies of stereoacuity and contrast

The effects of luminance contrast on stereoacuity have been investigated in a number of studies (Halpern & Blake, 1988; Legge & Gu, 1989; Heckmann & Schor, 1989; Cormack, Stevenson, & Schor, 1991; Simmons, 1992; Simmons & Hawken, 1993; Kontsevich & Tyler, 1994; Hess & Wilcox, 1994; Kingdom & Simmons, 1996; Wilcox & Hess, 1998), one of which additionally studied the effects of chromatic contrast (Kingdom & Simmons, 1996). The consensus of these studies is that stereoacuity improves with increasing chromatic or luminance contrast, providing the stimulus is processed by "first-order" stereopsis mechanisms [2] (Kingdom & Simmons, 1996; Wilcox & Hess, 1998). There is disagreement over the exact slope of the dependence, but a power-law relationship with a fractional exponent normally provides a reasonable fit for intermediate contrasts. The novel approach taken in this study is to incorporate the behaviour at extreme contrasts into the fitting process by allowing the fitted curve to asymptote (see Eq. (1)). This provides a more complete description of the data without having to resort to the arbitrary "kneepoint" determination that is a necessary component of piecewise linear fits.

Despite this new approach, and the different stimulus conditions used (successive presentation with feedback for 500 ms rather than a single presentation without feedback for 200 ms, and a slightly different threshold criterion), there is good qualitative agreement between the contrast dependences of stereoacuity in this study and in Kingdom and Simmons (1996), where the same two subjects were used. As in Kingdom and Simmons (1996), so in this study the slope parameters were similar for both chromatic (1.10) and luminance (1.15) contrast data with DS, but for FK, the slope parameters were shallower for colour (0.55) than for luminance (2.0). The

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[2] In the context of this experiment, "first-order stereopsis" refers to stereopsis dependent on the Gabor carrier information, and "second-order stereopsis" that dependent on the envelope information.

functional significance of this difference is not clear, although it may be related to the fact that FK has substantially better stereoacuity than DS under all comparable conditions. [3]

4.2. *Interactions between chromatic- and luminance-contrast-sensitive stereopsis mechanisms?*

It has been established in a previous study (Simmons & Kingdom, 1997) that we appear to have at least two stereopsis mechanisms, one sensitive to chromatic contrast and the other sensitive to luminance contrast, that process the sign of stereoscopic depth independently. Earlier studies have indicated that (a) the peak of the disparity tuning function for both mechanisms is the same (at least for a 0.5 cpd Gabor patch); (b) the contrast sensitivity of the chromatic-contrast-sensitive mechanism is lower, relative to detection threshold (Simmons & Kingdom, 1994); (c) the disparity range of the luminance-contrast-sensitive mechanism is larger, although this may be due to the absence of a second-order chromatic stereopsis mechanism (Simmons & Kingdom, 1995), and (d) the chromatic-contrast-sensitive mechanism has poorer stereoacuity for a given level of contrast above detection threshold (Kingdom & Simmons, 1996). Despite these differences, however, we were unaware perceptually of there being two separate depth sensations in the stimuli used in this study, one based on luminance contrast and the other based on chromatic contrast. It was hypothesized, therefore, that these mechanisms must interact *before* the extraction of stereoscopic depth.

Evidence for this interaction would be that an isoluminant chromatic stimulus, when superimposed on a fixed luminance contrast stimulus, would significantly affect stereoacuity and vice-versa. Just such evidence is presented in Fig. 3, where anti-correlated chromatic contrast severely disrupted stereopsis with a luminance-defined stimulus and in Fig. 4, where anti-correlated luminance contrast severely disrupted stereopsis with a colour-defined stimulus.

Does this interaction work both ways? That is, if anti-correlations disrupt stereopsis do correlations enhance it? The evidence from Figs. 3 and 4 is somewhat equivocal. There is a trend towards improved stereoacuity as the contrast of the correlated component increases, but improvements are slight and unsystematic. However, Figs. 5 and 6 show clearly that if performance with the luminance contrast component alone is poor then added correlated isoluminant chromatic contrast does improve it significantly.

It is thus clear that there are both negative and positive interactions between superimposed isoluminant and isochromatic stereoscopic stimuli. In order to quantify this putative interaction it is informative to replot the data from Figs. 3–5 with the appropriate contrast dependence data superimposed. The outcome of this exercise is shown in Fig. 7.

Looking at the top two panels of Fig. 7 (Fig. 3 with superimposed colour contrast dependence data) it is clear why no improvement was found with superimposed correlated colour contrast at these higher fixed luminance contrast levels: stereoacuity with the colour contrast component would never be good enough on its own to improve performance. It seems therefore that

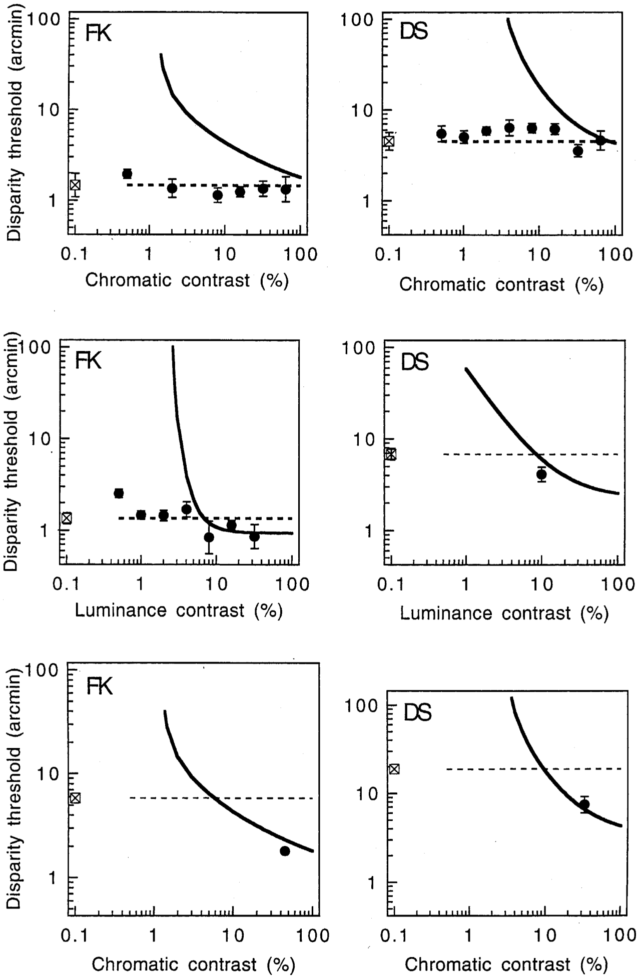

Fig. 7. Data from Figs. 3–5 replotted with the appropriate stereoacuity contrast dependencies from Fig. 2 superimposed. Top two panels are Fig. 3 with chromatic contrast dependence data superimposed. Middle two panels are Fig. 4 with luminance contrast dependence data superimposed. Bottom panel is Fig. 5 with chromatic contrast dependence data superimposed.

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[3] An anonymous referee pointed out that the luminance contrast dependence for FK in this study actually looked rather shallow with a steep asymptote at low contrast. In Fig. 3 of Kingdom and Simmons (1996) the slope for the complementary condition was determined over a relatively narrow range of contrasts and the disparity threshold reached an asymptote at a luminance contrast of about 10%. This contrast dependence is quite similar to that shown in Fig. 2 of this study.

these subjects relied entirely on the luminance contrast component whatever the level of superimposed correlated colour contrast. In the middle two panels of Fig. 7 (Fig. 4 with the luminance contrast dependence data superimposed), the slight improvements found with added correlated luminance contrast seem to be consistent with subjects switching from a reliance on the colour contrast component to a reliance on the luminance contrast component when the latter was providing more precise information. The bottom two panels of Fig. 7 (Fig. 5 with the appropriate colour contrast dependence data superimposed) shows that this effect is also apparent with superimposed colour contrast, provided the luminance contrast component is low enough.

This analysis suggests that the interactions between chromatic and achromatic stereopsis mechanisms are essentially on a simple "winner-take-all" basis. Somehow the visual system can determine which of the two stimulus components is giving the more precise information and uses that as the basis for its depth judgements. However, this idea is not consistent with the data from Fig. 6 in which performance with the fixed high colour contrast stimulus is unaffected by superimposed luminance contrast despite the potentially increased precision provided by the high luminance contrast component.

The simple "winner-take-all" interaction is also hard to square with the data obtained using anti-correlated stimuli. With these stimuli the disruption of stereoacuity seems to happen at quite low anti-correlated contrasts, before the stereoacuity of the variable component is even close to that of the reference. The effect of the anti-correlated contrast should be to give the stimulus a large disparity of opposite sign to that intended. [4] The effect of this additional depth signal could vary with disparity because, when the actual stimulus disparity is small (e.g. $+5'$) the anti-correlated disparity would be large ($-55'$ in this example) and possibly less disruptive than when the stimulus disparity was larger. This could complicate the effects of responses on the convergence of a staircase routine. Another interesting point to note about the anti-correlated data in Figs. 3 and 4 is that they appear to level out at a disparity threshold of around $20'$ for both subjects. This may be because second-order stereopsis mechanisms (which will give consistent information with the reference contrast, regardless of whether the stimulus is correlated or anti-correlated) are providing a stereoscopic "safety net". When this safety net was weakened during the low-luminance-contrast reference experiment (Fig. 5), stereoacuity was much more substantially disrupted (disparity thresholds were greater than 1°).

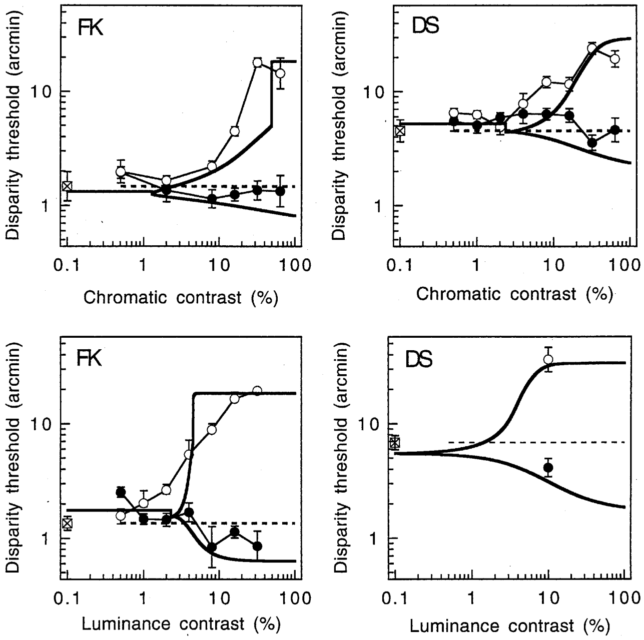

Fig. 8. Data from Figs. 3 and 4 replotted again with superimposed predicted stereoacuities obtained using the probability summation model outlined in the Appendix.

4.3. Interaction by probability summation between independent mechanisms?

Simmons and Kingdom (1997) used a probability summation model to explain performance in a stereoscopic depth task where the stimuli contained both chromatic and achromatic stereoscopic components. Could a similar model be applied to the data obtained in this study? Fig. 8 shows the predictions of just such a model applied to the data illustrated in Figs. 3 and 4. The modelling is relatively complex, so further details are given in an Appendix. The principle, however, is that the subjects' responses are based on the outcome of probability summation between three independent stereopsis mechanisms: luminance-contrast-sensitive first-order and second-order and chromatic-contrast-sensitive first-order. [5] For each combination of luminance contrast, chromatic contrast and disparity it is possible to determine the probability of a "front" or "behind" response. Baseline data for this comparison is taken from Fig. 2 (although the second-order luminance mechanism had to be estimated, as it was not explicitly measured in the experiments). This probability distribution was used to generate psychometric functions for the task, which allowed a threshold performance level to be determined.

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[4] The disparity of the anti-correlated component would be $p/2 - d$, where p is the period of the carrier and d the intended stimulus disparity, assuming matching by nearest neighbour of similar brightness or chromatic polarity.

[5] Previous results suggested that the second-order chromatic stereopsis mechanism was either very weak or non-existent (Simmons & Kingdom, 1995; Kingdom & Simmons, 1996).

The data are re-plotted with predictions from the probability summation model in Fig. 8. The fit to the data is reasonably good in qualitative terms, although there is a tendency for the model to over-estimate the improvement in stereoacuity with high correlated contrasts. Note that the only variable parameter in the fits were the stereoacuities of the second-order luminance mechanism. This mechanism was necessary to explain why the task did not become impossible with high levels of anti-correlated contrast. It was also assumed that the stereoacuity of this second-order luminance mechanism did not vary with contrast (Wilcox & Hess, 1998) and that no variable weighting is applied to the contributions of any stereopsis mechanism to the final judgement. Our hypothesis is that if we were to "tinker" with the model in these plausible ways then the fit to the data could be made even better.

4.4. Feature matching or mechanism interaction?

Our suggestion is, then, that the pattern of results in this study is consistent with the encoding of stereoscopic depth information by at least two, and probably three stereopsis mechanisms: first-order and second-order luminance stereopsis and first-order chromatic stereopsis. These mechanisms interact to provide a unified depth percept. There is no need to invoke a high-level chromatic feature matching mechanism to explain the results obtained in this study, although the results are not necessarily inconsistent with such a mechanism. Future psychophysical and computational investigations will be necessary to distinguish between these two flavours of explanation and also to determine the precise rules which govern the interactions between our putative multiple stereopsis mechanisms.

Acknowledgements

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Appendix A. Probability summation of depth sign judgements between independent stereopsis mechanisms

It was assumed that performance in this depth discrimination task was based on the output states of three independent stereopsis mechanisms: first-order luminance, second-order luminance and first-order colour. It was further assumed that the probabilities of these different states summate to produce an overall state which determines the observer's response to the stimulus.

In the task a stereoscopic stimulus was presented in both intervals of a 2IFC. In one of those intervals the stimulus was given crossed disparity relative to the fixation circle, and in the other the stimulus was given uncrossed disparity relative to the fixation circle. For modelling purposes let us assume that all presentations are "Front first, Behind second", so that the correct response is "first interval". Let F_1, N_1, B_1 represent the states "Front, Neutral, Behind" in the first interval and F_2, N_2, B_2 the same for the second interval. Let $P(X)$ be the probability of a particular state. The set, U, of possible states is given by the ordered pairs:

$$U = \{(F_1,F_2),(F_1,N_2),(F_1,B_2),(N_1,F_2),(N_1,N_2),(N_1,B_2),\\ (B_1,F_2),(B_1,N_2),(B_1,B_2)\}$$

The subject will respond correctly for the states $F = \{(F_1,N_2),(F_1,B_2),(N_1,B_2)\}$ and incorrectly for the states $B = \{(N_1,F_2),(B_1,F_2),(B_1,N_2)\}$. For the three neutral states $N = \{(F_1,F_2),(N_1,N_2),(B_1,B_2)\}$ the subject will guess and therefore be correct 50% of the time. This can be summarised as:

$$P(\text{correct}) = P(F) + 0.5P(N) \tag{A.1}$$

In general, $P(F)$, $P(N)$ and $P(B)$ will vary with the contrast and disparity of the stereoscopic stimulus. If the dependencies of these probabilities on contrast and disparity are known (or can be reliably estimated), then P(correct) can be calculated for any disparity-contrast combination.

Let us first look at the case for a single stereoscopic stimulus. Let us assume that both intervals are independent, such that $P((X,Y)) = P(X)P(Y)$, therefore, from the set F:

$$P(F) = P(F_1)P(B_2) + P(F_1)P(N_2) + P(N_1)P(B_2) \tag{A.2}$$

Now,

$$P(F_1) + P(N_1) + P(B_1) = 1 \tag{A.3}$$

But, if we assume that stereoscopic matching is perfect, $P(B_1) = 0$.

Therefore,

$$P(N_1) = 1 - P(F_1) \tag{A.4}$$

Assuming no biases, then, by symmetry,

$$P(B_2) = P(F_1) \tag{A.5}$$

$$P(N_2) = P(N_1) \tag{A.6}$$

and

$$P(F_2) = 0 \tag{A.7}$$

Hence, by substitution into (A.2):

$$P(F) = (P(F_1))^2 + P(F_1)(1 - P(F_1)) + (1 - P(F_1))P(F_1) \tag{A.8}$$

Now,

$$P(N) = P(F_1)P(F_2) + P(N_1)P(N_2) + P(B_1)P(B_2) \tag{A.9}$$

But, from (A.4) and (A.7), $P(B_1) = P(F_2) = 0$, so

$$P(N) = P(N_1)P(N_2) = (1 - P(F_1))^2 \tag{A.10}$$

This then allows (A.1) to be entirely re-expressed in terms of $P(F_1)$. For clarity, let $P(F_1) = X$, then:

$$P(\text{correct}) = X^2 + 2X(1 - X) + 0.5(1 - X)^2 \tag{A.11}$$

(A.11) becomes a quadratic expression in X:

$$P(\text{correct}) = -0.5X^2 + X + 0.5 \tag{A.12}$$

Let $P(\text{correct}) = p(0.5 \leqslant p \leqslant 1)$, then this becomes a quadratic equation with the solution:

$$X = 1 \pm \sqrt{2(1 - p)} \tag{A.13}$$

where only the solution where $0 \leqslant X \leqslant 1$ is appropriate. Hence if p is known as a function of contrast and disparity, then so is X.

A.1. Determining p as a function of contrast and disparity

The data presented in Fig. 2 of the main paper give the contrast dependencies for stereoacuity in this task as a function of chromatic or luminance contrast for the two subjects. The expression in Eq. (1) of the main text, when given appropriate parameters, provides a fit to these data, which gives the disparity at which $p = 0.707$ for a range of chromatic and luminance contrasts. It is well known that the disparity dependence of proportion-correct performance in a stereoscopic discrimination task is well modelled by a cumulative Gaussian psychometric function. By setting up appropriate such psychometric functions to intersect with the contrast-dependence data, a probability surface was calculated.

Values of p could be sampled from this surface, allowing the determination of X (i.e. $P(F_1)$) for all values of contrast and disparity for both chromatic and luminance contrast (first-order). It was assumed that disparity thresholds for second-order luminance-based stereopsis were constant at n-times the minimum disparity threshold obtained with the luminance stimulus (i.e. $n \times \delta_L$). [6] This is a suitable range for stereoacuity with contrast envelope stimuli (see, e.g., Wilcox & Hess, 1998).

A.2. Predictions for compound stimuli

When all three stereopsis mechanisms are contributing to the perceptual judgement, the problem becomes one of determining their consensus state. Again, let us assume that each mechanism has three possible states: F, N and B. Let us assume that each mechanism contributes with equal weight. The contributions from each mechanism will be subscripted L, C and E to signify luminance (first-order), colour (first-order) and envelope (luminance second-order) mechanisms respectively.

An overall "front" state in, say, the first interval of a trial will result from the following combinations:

$$F_1 = \{(F_L, F_C, F_E), (F_L, F_C, N_E), (F_L, F_C, B_E), (F_L, N_C, F_E), (F_L, B_C, F_E), (N_L, F_C, F_E), (B_L, F_C, F_E), (F_L, N_C, N_E), (N_L, F_C, N_E), (N_L, N_C, F_E)\}.$$

Note the assumption that, if all mechanisms are neutral apart from one, that response will follow the non-neutral mechanism (i.e. subjects favour depth over flatness). The combinations giving a "Behind" state are, by symmetry:

$$B_1 = \{(B_L, B_C, B_E), (B_L, B_C, N_E), (B_L, B_C, F_E), (B_L, N_C, B_E), (B_L, F_C, B_E), (N_L, B_C, B_E), (F_L, B_C, B_E), (B_L, N_C, N_E), (N_L, B_C, N_E), (N_L, N_C, B_E)\}.$$

Because of the depth-over-flatness assumption, the neutral state set N_1 has fewer members:

$$N_1 = \{(N_L, N_C, N_E), (F_L, N_C, B_E), (F_L, B_C, N_E), (N_L, F_C, B_E), (N_L, B_C, F_E), (B_L, N_C, F_E), (B_L, F_C, N_E)\}.$$

Making the assumption that each mechanism is independent means that we can calculate the probabilities of each of these sub-states and hence the probability of the overall state for a given stimulus or stimulus combination. We assume that the individual mechanism will only give "veridical" responses, such that a stimulus with crossed disparity, relative to fixation, will only give "front" or "neutral" states.

A.3. Predictions for correlated stimuli

When the chromatic and luminance information is correlated all three mechanisms will signal the same disparity sign, but as their stereoacuities differ from each other and as a function of contrast the overall stereoacuity should sometimes be better than if a single mechanism was controlling performance.

Our goal is to find threshold performance for the task, i.e. $p(\text{correct}) = 0.707$. From (A.1) we therefore need to find $P(F)$ and $P(N)$ for the task. From (A.2) this reduces to finding $P(F_1)$, $P(F_2)$, $P(N_1)$, $P(N_2)$, $P(B_1)$ and $P(B_2)$ for the task. Again, for modelling purposes we assume that all trials are "front first, behind second", and, that, as the mechanisms are receiving correlated

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[6] The value of n was 20 for FK and 10 for DS.

information, that (A.3)–(A.10) also apply. Hence, by finding $P(F_1)$ in terms of the probabilities of different states in the three mechanisms, the problem will have been solved. For notational simplicity, let $P(F_L) = L$, $P(F_C) = C$ and $P(F_E) = E$.

Assigning probabilities to each element of the set F_1 given above:

$$\begin{aligned} P(F_1) = {} & LCE + LC(1-E) + 0 + L(1-C)E + 0 \\ & + (1-L)CE + 0 + L(1-C)(1-E) \\ & + (1-L)C(1-E) + (1-L)(1-C)E \end{aligned} \tag{A.14}$$

This expression simplifies to:

$$P(F_1) = LCE + L(1-C) + C(1-E) + E(1-L) \tag{A.15}$$

For $p = 0.707$, from Eq. (A.13), at threshold $P(F_1) = 0.2345$. Finding the disparity at which this is true for a given combination of luminance and colour contrasts allowed the predictions to be plotted (see Fig. 8).

A.4. Predictions for anti-correlated stimuli

The predictions for anti-correlated stimuli are more complicated, as one of the assumptions made in all the above calculations is untrue, namely $P(B_1) \neq 0$. Working as before, by assigning probabilities to the sets F_1, N_1 and B_1 but, assuming that we are dealing with a luminance stimulus with anti-correlated colour information, we shall make $P(F_L) = L$ and $P(F_E) = E$, but $C' = P(B_C)$. We also assume that $P(F_C) = 0$, Hence:

$$\begin{aligned} P(F_1) = {} & 0 + 0 + 0 + L(1-C')E + LC'E + 0 + 0 \\ & + L(1-C')(1-E) + 0 + (1-L)(1-C')E \end{aligned} \tag{A.16}$$

This simplifies to:

$$P(F_1) = 2LC'E + L(1-C') + E(1-L-C') \tag{A.17}$$

Using the same methodology we find that:

$$\begin{aligned} P(N_1) = {} & (1-L)(1-C')(1-E) + 0 + LC'(1-E) \\ & + 0 + (1-L)C'E + 0 + 0 \end{aligned} \tag{A.18}$$

which simplifies to:

$$\begin{aligned} P(N_1) = {} & 1 - 3LC'E - L(1-2C') - C'(1-2E) \\ & - E(1-L-C') \end{aligned} \tag{A.19}$$

(A.5) and (A.6) still apply, so (A.2) becomes:

$$P(F) = (P(F_1))^2 + 2P(F_1)P(N_1) \tag{A.20}$$

and

$$P(N) = (P(N_1))^2 \tag{A.21}$$

Consequently, by substitution into (A.1):

$$P(\text{correct}) = (P(F_1))^2 + 2P(F_1)P(N_1) + 0.5(P(N_1))^2 \tag{A.22}$$

By substituting (A.17) and (A.19) into (A.22), values of L, C' and E can be found which satisfy $P(\text{correct}) = 0.707$ and the contrast dependencies for disparity thresholds with anti-correlated stimuli can be predicted (see Fig. 8).

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