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# The missing-fundamental illusion at isoluminance

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Frederick A A Kingdom

McGill Vision Research, Department of Ophthalmology, McGill University, 687 Pine Avenue West, H4-14, Montréal, Québec H3A 1A1, Canada; e-mail: fred@jiffy.vision.mcgill.ca

David R Simmons

Department of Vision Sciences, Glasgow Caledonian University, City Campus, Cowcaddens Road, Glasgow G4 OB4, Scotland, UK

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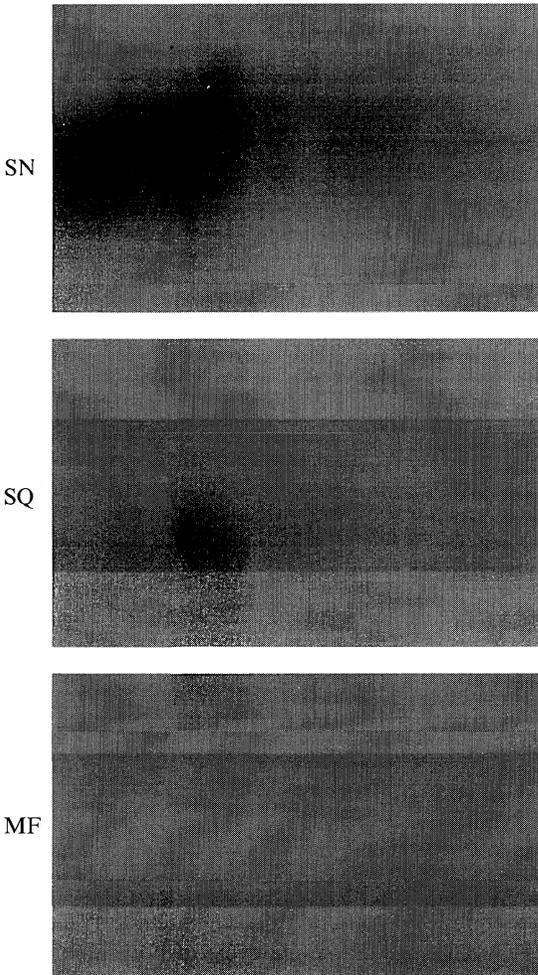
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**Abstract.** The missing-fundamental illusion describes how a square wave with its fundamental Fourier component removed appears as a square wave. This illusion is normally explained with reference to the bandpass nature of the luminance-contrast-sensitivity function, together with a 'default-to-square-wave' rule. Since the chromatic-contrast-sensitivity function is low-pass, we should not expect a missing-fundamental illusion at isoluminance. Using a simultaneous-detection-and-identification paradigm to eliminate contrast as a cue to discrimination, we nevertheless found that chromatic missing fundamentals and square waves could not be separately identified at detection threshold: just under twice the contrast required to detect the stimuli was needed to identify them. To test whether this was due to insufficiently narrow chromatic-channel bandwidths, we measured detection and identification thresholds for chromatic  $F$  and  $3F$  sine-wave gratings. In this case identification was possible almost at detection threshold, suggesting that channel bandwidth limitations were not the critical factor. It is suggested that the weak missing-fundamental illusion observed at isoluminance probably reflects the operation of mechanisms similar to those that are responsible for the chromatic Craik–Cornsweet illusion.

## 1 Introduction

The missing-fundamental illusion describes how a square wave with its fundamental Fourier component removed appears as a square wave (Campbell et al 1971, 1978; see review by Kingdom and Moulden 1988). Figure 1 shows a single cycle of both a square-wave (SQ) and a missing-fundamental (MF) stimulus. At close viewing distances the central bars of the SQ and MF stimuli should be indistinguishable. It is widely accepted that this illusion is in part due to the decline in luminance-contrast sensitivity at low spatial frequencies. The relative insensitivity to the fundamental component of the square wave results in near-identical neural images for missing fundamentals and square waves at low contrast, low spatial frequencies. Under these circumstances the visual system ascribes the appearance of a square wave to the missing-fundamental stimulus [termed the 'default-to-square-wave' rule by Campbell et al (1971)].

The specific conditions under which the illusion occurs have been explained in terms of linear channel theory. The presence of narrow-band spatial channels operating independently at threshold predicts that the contrast threshold for discriminating a square wave from a missing fundamental will be the same as the contrast threshold for detecting the fundamental when presented alone (after taking into account the  $4/\pi$  amplitude ratio of the fundamental to its parent square). This may be seen in figure 1, which also shows the fundamental component [the sine-wave (SN) stimulus], of the square wave (SQ). The MF stimulus in figure 1 is constructed by subtracting the SN from the SQ stimulus. As figure 1 is brought closer to the viewer, the point where the SQ and MF stimuli become indistinguishable should also be the point where the SN stimulus disappears. This observation has been upheld empirically for both periodic



**Figure 1.** Black-and-white versions of the 'dark-bar' luminance sine-wave (SN), square-wave (SQ), and missing-fundamental (MF) stimuli used in the experiments. The luminance profiles of the three stimuli are shown on the right-hand side of figure 2. The MF stimulus is constructed by subtracting the SN stimulus from the SQ stimulus. If viewed from afar, the central bars of the SQ and MF stimuli will be discriminable, and the SN bar detectable. As the page is brought closer to the viewer, however, a point should be reached where the SQ and MF stimuli become indistinguishable and the SN bar disappears. Tilting the page away from the viewer has the same effect as increasing viewing distance.

(Campbell et al 1971, 1978; though see Sullivan and Georgeson 1977, where this prediction was not so well met) and aperiodic (Burr 1987; Kingdom 1996) stimuli.

The chromatic-contrast-sensitivity function is low-pass (Granger and Hurtley 1973; Mullen 1985), and like its luminance counterpart is believed to reflect the umbrella of a number of independent spatial channels (Losada and Mullen 1994). It follows therefore that at isoluminance no spatial frequency of square wave exists at which the higher harmonics in the stimulus (which on their own define the missing fundamental) will be more detectable than the fundamental. Thus there are no situations in which the neural images of an isoluminous square wave and missing fundamental will be identical, and therefore no situations in which they will be perceptually indistinguishable. At isoluminance, therefore, we should expect square waves and missing fundamentals always to be discriminable at detection threshold. The purpose of the experiments reported here was to test this prediction.

## 2 Methods

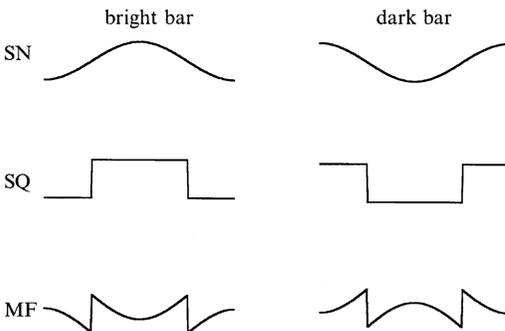
### 2.1 Apparatus

The stimuli were generated with the VSG2/1 Digital Signal Generator (DSP) (Cambridge Research Systems) hosted by a Dell 386 computer and displayed on a Barco CDCT 6551 RGB monitor. The VSG2/1 DSP generates stimuli with three 12-bit (4096 levels) lookup

tables, one for each RGB channel, each selected from 14-bit DACs (digital-to-analog converters). Calibration to produce  $Z$ -linearised lookup tables for the red and green channels was performed with a UDT photometer. The photometer was centred on the middle bar of a red or green modulated square wave that was identical in profile to that employed in the actual experiments. The bar was displayed at a range of luminances.

## 2.2 Stimuli

Sine (SN), square (SQ), and missing-fundamental (MF) waveforms were employed. Figure 1 shows black-and-white versions of the dark-bar luminance stimuli, whose luminance profiles are shown in figure 2 along with their bright-bar counterpart profiles. All waveforms were vertically modulated with a spatial frequency of 1 cycle per screen, and were presented in cosine phase. As figure 1 shows, the subject saw a single bar of the stimulus in the middle of the screen. Because the experiments were conducted in a darkened room, the whole display was essentially on a black background.<sup>(1)</sup> Screen display size was 26 deg (width)  $\times$  16 deg (height) at the middle viewing distance of 74 cm. For the luminance stimuli, the sine-wave profile was given by  $L(x) = M + A \cos(2\pi fx + \phi)$ , where  $M$  is mean luminance of  $32 \text{ cd m}^{-2}$ ,  $A$  amplitude,  $f$  spatial frequency, and  $\phi$  phase. The different spatial frequencies were achieved by changing viewing distance, and the three viewing distances employed were 37, 74, and 148 cm. This resulted in spatial frequencies of 0.031, 0.063, and 0.125 cycle  $\text{deg}^{-1}$  respectively. These very low spatial frequencies were chosen to give the best chance for the missing-fundamental illusion to manifest itself at isoluminance. The phase  $\phi$  was set to either 0 or  $\pi$  radians, producing a central bright, or dark, bar in the case of the luminance stimuli, and a central red, or green, bar respectively in the case of the chromatic stimuli. The missing fundamental was obtained by subtracting a sine wave of amplitude  $4/\pi$  from a square wave of unit amplitude. Thus, in figure 1, the bright-bar MF stimulus equals the bright-bar SQ stimulus minus the bright-bar SN stimulus ( $\times 4/\pi$ ), and similarly for the dark-bar stimuli. In all experiments except the measurement of isoluminance, the stimuli were presented for 0.3 s with a 0.1 s ramp at the beginning and end of presentation.



**Figure 2.** Profiles of the luminance stimuli used in the experiments. The dark-bar luminance profiles on the right correspond to the stimuli shown in figure 1.

<sup>(1)</sup> The stimuli were not Gaussian-enveloped, and therefore at the edges of the display there would be a small difference between the luminance/colour of the SQ, and the SN and MF stimuli (see figure 1). However, since the stimulus contrasts employed in the experiment were near threshold, any differences between the luminance/colour of the SQ and the other two waveforms at the edge of the display would be very small. Moreover, the differences would be masked by the high-contrast edge of the stimulus with its black surround; and, because subjects always fixated the middle of the screen, we assumed that they would not form the basis for discrimination of the waveforms. A reviewer suggested we test this assumption by measuring discrimination thresholds with the display occluded except for its 2 cm outer annulus. We did this for the isoluminance SQ versus MF 0.063 cycle  $\text{deg}^{-1}$  condition, using FK and a naive observer, HCL, as subjects. We found that contrast thresholds for discriminating the isoluminous SQ from MF stimuli were elevated by a factor of 9.4 for subject FK and 6.0 for subject HCL when only the annulus was displayed. This confirms that our assumption was correct.

Only the red and green guns of the monitor were employed. The CIE chromaticity coordinates were  $x = 0.60$ ,  $y = 0.35$  for the red phosphor; and  $x = 0.28$ ,  $y = 0.6$  for the green phosphor. These were modulated in phase for the yellow–black stimuli, and in antiphase for the red–green stimuli. Contrast sensitivity was defined as the reciprocal of Michelson contrast:  $(L_{\max} - L_{\min}) / (L_{\max} + L_{\min})$  at threshold.

### 2.3 Subjects

Three subjects were used for the main experiment: FK and DS (the authors), and SB, who was a paid undergraduate volunteer. SB was an experienced psychophysical observer but naive as to the purpose of the experiment. In addition, HCL, a postdoctoral fellow and also naive to the purpose of the experiment, performed a control experiment (see footnote). Subjects FK and HCL were emmetropic, while DS and SB used their optical corrections, except for SB at the closest viewing distance.

### 2.4 Procedure

**2.4.1 Determination of isoluminance point.** Isoluminance was achieved by using the method of minimum motion (Moreland 1982; Anstis and Cavanagh 1983). A red–green sine wave of spatial frequency 3 cycles per screen and contrast of 0.1 was drifted across the screen at approximately 1 Hz. The spatial frequency of the sine wave was chosen to be equal to that of the third harmonic component of the SQ and MF stimuli, and isoluminance was measured at each of the three viewing distances employed in the main experiment. The task for the subject was to adjust the  $R/(R + G)$  ratio of the stimulus until a minimum motion percept was achieved. Between 20 and 30 measurements were taken at each viewing distance and averaged to give the isoluminance point. The isoluminance  $R/(R + G)$  ratio for subject FK was 0.44 for all three viewing distances. For subject SB it was 0.48, 0.48, and 0.47; and for subject DS 0.52, 0.51, and 0.50 for the 0.031, 0.063, and 0.125 cycle  $\text{deg}^{-1}$  conditions, respectively. For subject HCL the isoluminance  $R/(R + G)$  ratio was 0.51 for the 0.063 cycle  $\text{deg}^{-1}$  condition employed in the control experiment (see footnote). In the conditions involving chromatic stimuli only these isoluminance  $R/(R + G)$  ratios were used. Since contrast thresholds were measured throughout, we assume that any luminance artifacts introduced by a task having a slightly different isoluminance point from that defined by minimum motion, or due to the minimum motion estimates being slightly in error, are tiny.

**2.4.2 Separate detection and discrimination.** In the first experiment a two-interval forced-choice procedure was used to measure contrast thresholds for detecting SN, SQ, and MF stimuli, as well as contrast thresholds for discriminating SQ from MF stimuli. In the detection task one stimulus was presented on each trial and the subject was required to indicate by button press in which interval it appeared. In the discrimination task two stimuli, a SQ and a MF of the same contrast, were presented in the two intervals, and the subject indicated the interval with the SQ stimulus. For both tasks a standard two-up, one-down staircase procedure established the 70.7%-correct level (Levitt 1971). A tone for an incorrect response was provided as feedback. The experiment was terminated after ten reversals of the staircase, with the contrast threshold calculated as the geometric mean contrast over the last eight reversals.

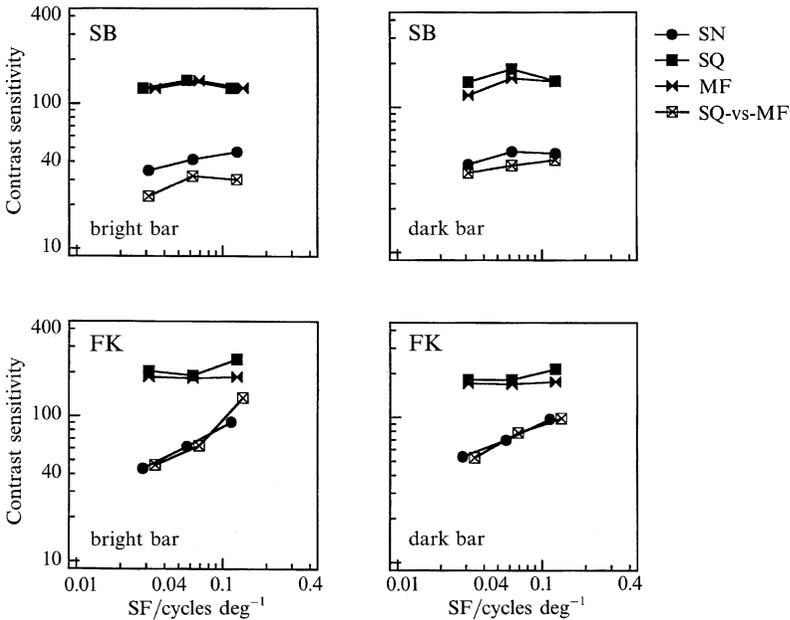
**2.4.3 Simultaneous detection and identification.** In the second and third experiments, a simultaneous-detection-and-identification procedure was employed, with the method of constant stimuli. On each trial there were two intervals, and in one interval one of two stimuli (in experiment 2, SQ or MF; in experiment 3,  $F$  or  $3F$  sine waves) was presented at one of a range of contrasts. The subject was required to make two responses: the first to indicate in which interval the stimulus was present, the second to indicate which type of stimulus was present. A tone for an incorrect decision was given immediately after each response. Nine contrast levels were employed for each condition,

set at intervals appropriate to produce both detection and identification psychometric functions as determined by pilot studies. A total of 720 trials were conducted for each condition (red–green, yellow–black, each at the three viewing distances), which meant that for each stimulus the detection and identification psychometric functions were based on 360 trials. The method used to analyse the data will be described later.

### 3 Experiment 1. Detection and discrimination thresholds for SN, SQ, and MF

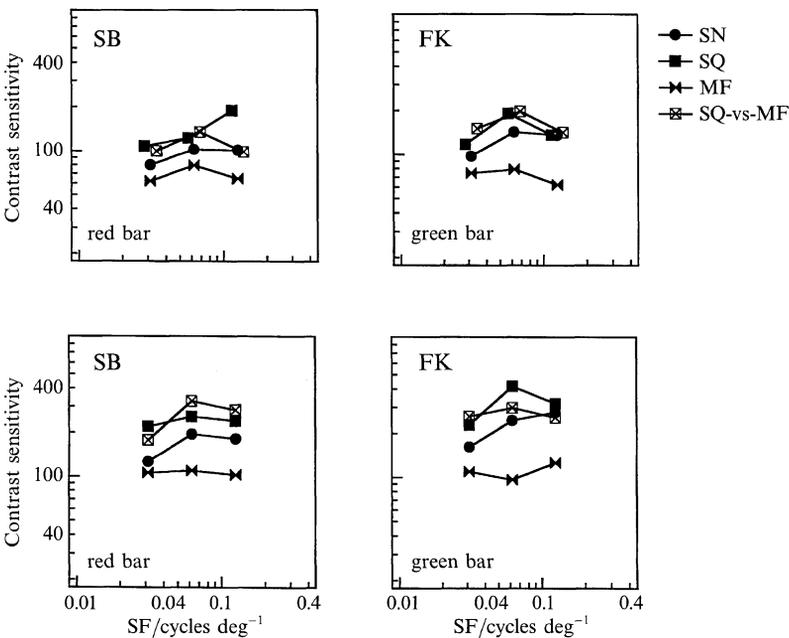
In the first experiment we measured contrast thresholds for detecting SN, SQ, and MF stimuli, and discriminating SQ from equal-contrast MF stimuli. The results are shown in figure 3 for the isochromatic yellow–black stimuli, and in figure 4 for the isoluminous red–green stimuli. In figure 3 the isochromatic data clearly fall into two groups, with similar sensitivities to the SQ and MF stimuli (both dependent on the third harmonic) and similar but lower sensitivities to the SN stimulus and to SQ-versus-MF discrimination (both dependent on the fundamental). These results are similar to those obtained in previous studies (Campbell et al 1978; Burr 1987; Kingdom 1996). A measure of the magnitude of the missing-fundamental illusion is the ratio of the (geometric) mean of the SQ and MF detection thresholds to the SQ-versus-MF discrimination thresholds. This ratio ranged from about 4.5 at the lowest spatial frequency to 2.8 at the highest spatial frequency. According to Campbell et al’s (1978) model we should predict that MF-versus-SQ thresholds equal SN thresholds  $\times 4/\pi$ . If we take the ratio of predicted and measured MF-versus-SQ thresholds, and average across conditions, we obtain 1.7 for subject SB and 1.2 for subject FK. This represents a reasonably good fit to the model, as was found in a recent study with similar stimuli (Kingdom 1996).

The results for the isoluminous red–green stimuli in figure 4 are very different. Highest sensitivity is to SQ and SQ-versus-MF, then to SN, and lowest to MF. A feature of the data first worth noting is that the SQ thresholds are nicely predicted from the SN



**Figure 3.** Results for the isochromatic yellow–black stimuli in experiment 1. The symbols for SN, SQ, and MF are detection thresholds for sine-wave, square-wave, and missing-fundamental-wave stimuli, respectively. The symbol for SQ-versus-MF represents the contrast discrimination threshold for discriminating a SQ stimulus from an equal-contrast MF stimulus. For clarity, standard errors, which were typically the height of the symbols, are not shown.

thresholds, on the assumption that the detectability of the SQ stimulus is based on its fundamental. Multiplying the SN thresholds by  $4/\pi$  and dividing the result by the SQ thresholds gives ratios of 1.02 for subject SB and 1.12 for subject FK, a very close prediction for both subjects. The MF-versus-SQ thresholds appear almost identical to the SQ thresholds, showing that as soon as the SQ stimulus is detectable it is discriminable from an equal-contrast MF stimulus. This might be taken to imply an absence of a missing-fundamental illusion at isoluminance. However, such a conclusion would be premature, given that the MF stimuli are so much less detectable than the SQ stimuli. Under such circumstances the discrimination task can be accomplished simply by responding to the interval in which the stimulus is visible. Therefore these results do not tell us whether MF and SQ isoluminous stimuli are discriminable at threshold when contrast is not a cue, ie when it is the chromatic profile of the stimulus that must be discriminated. For this reason we performed the following experiment.



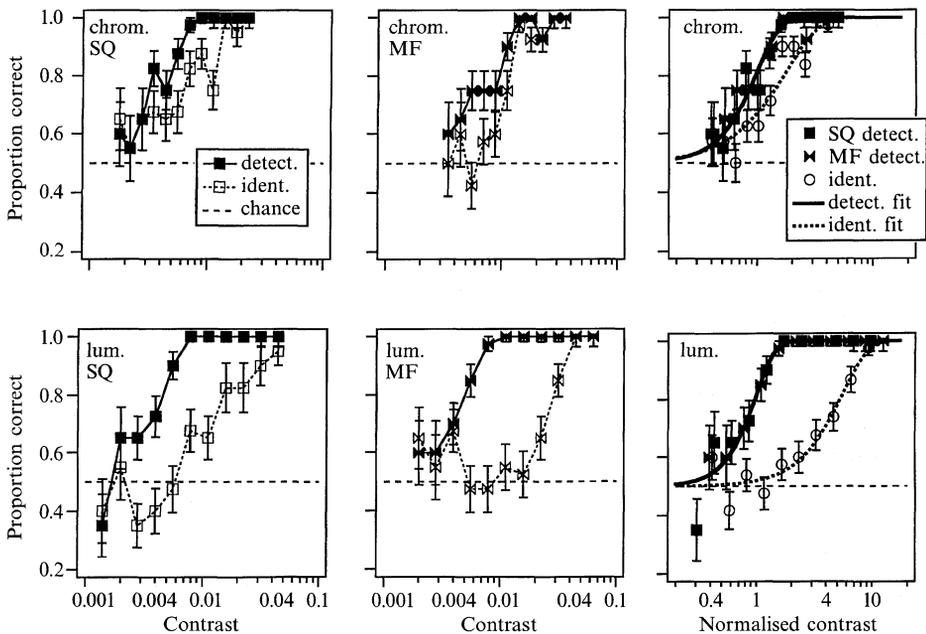
**Figure 4.** Results for the isoluminous red–green stimuli in experiment 1.

#### 4 Experiment 2. Simultaneous detection and identification of SQ and MF

In order to eliminate contrast as a cue to identifying which one of a forced-choice SQ and MF pair was the SQ stimulus, we employed a simultaneous-detection-and-identification paradigm (see section 2.4.3). The key feature of this technique is that only one of the pair-to-be-discriminated stimuli is presented on each trial, and at a contrast randomly selected from a predetermined set chosen to approximately equate the visibility of the two stimuli. This procedure eliminates contrast as a cue to identification. Psychometric functions were obtained for both detecting and identifying the SQ and MF stimuli. Each subject was given approximately 1000 trials practice to ensure familiarity with the stimuli.

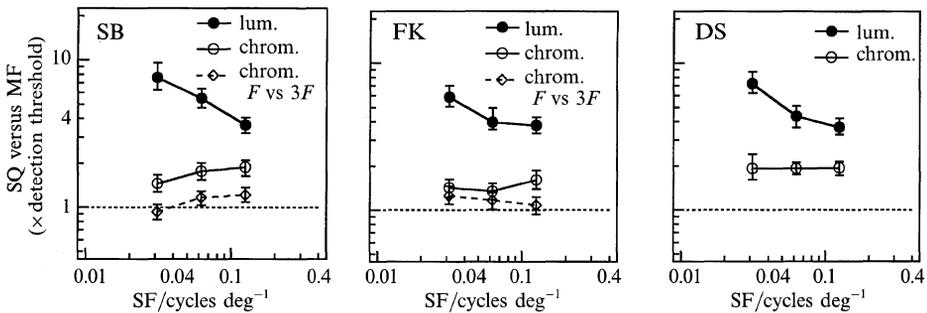
One of the shortcomings of the simultaneous-detection-and-identification paradigm is that the method is prone to influence from subject biases. These biases can occur because the identification task is not the same as a conventional discrimination task, where two stimuli are presented and the subject is asked to discriminate one from the other. If the subject is more inclined to identify, for example, low-contrast stimuli as square waves, then a significant bias would be evident in the identification data, with artifactually

different contrast thresholds for identification of the two types of stimuli. One way to get around this problem of identification bias is to combine the data from the two identification psychometric functions (Watson and Robson 1981). However, this is difficult if the contrast ranges spanned by the psychometric function data are different for the two stimuli, as is the case for the chromatic SQ and MF stimuli. Consequently we normalised the data by detection threshold, as it was apparent that such normalised data were similar for both stimuli. The combination was performed as follows. Detection contrast thresholds were determined for SQ and MF stimuli by fitting Weibull–Quick functions to the proportion-correct data. The threshold criterion for these functions was 81.6% correct. The fitting method used was similar to that described by Watson (1979). These detection thresholds were then used to calculate the contrast of the stimuli in multiples of detection threshold. If the contrasts to be combined were different, then a mean was taken (note that they could only differ by a maximum of half the contrast step size). These combined psychometric functions were then fitted by the same routines as those used to fit the detection functions. Furthermore, ‘bootstrap’ analysis was employed to generate confidence limits on the threshold values obtained (see Simmons and Kingdom 1994, for more details). The end result of this analysis was a contrast threshold for correct identification expressed in multiples of detection threshold, with associated confidence limits. In general, this threshold was approximately equal to that obtained by fitting each set of identification data separately and then taking the mean of the two thresholds obtained, but the chief advantages of combining the data were that biases were cancelled out and authentic confidence limits on the data could be obtained. An example set of psychometric functions which illustrates the method we employed is shown in figure 5.



**Figure 5.** Psychometric functions for detection and identification obtained in experiment 2 for the naive subject SB ( $0.063 \text{ cycle deg}^{-1}$  condition). The top row of graphs is for the chromatic condition, the bottom row for the luminance condition. Left and middle graphs: raw psychometric functions for SQ and MF. The right-hand graphs show the combined SQ plus MF data fitted with Weibull functions, as explained in the text. Error bars are the binomial standard errors calculated by assuming that each data point was a sample from a binomial distribution. The horizontal dashed line indicates chance performance level.

The results of this experiment are presented in figure 6 for three subjects: FK and SB, who performed the first experiment, and an additional subject, DS. Since in the first experiment there were no systematic differences between the results depending on the phase of the stimuli (dark versus bright bar, or red versus green bar), we used only the dark-bar and red-bar stimuli. The open and closed symbols represent the chromatic and luminance data, respectively, and show the size of the contrast gap between detection, given by the dotted zero line, and identification. The error bars are 95%-confidence limits as determined by the bootstrap analysis, and serve as the basis for determining whether one can reject the hypothesis that identification can be performed at detection threshold. It is clear from the data that for both the luminance and chromatic data this hypothesis can be rejected: we find that SQ versus MF identification is not possible at threshold for any condition. The size of the contrast gap is, however, much greater for the luminance than for the chromatic condition, and only the luminance condition shows a clear spatial-frequency dependence. For the luminance condition, geometric mean identification/detection ratios for the SQ and MF stimuli ranged from 7.1 to 4.0 depending on spatial frequency. For the chromatic stimuli the ratio was on average 1.8.



**Figure 6.** Results for experiments 2 and 3 (subjects SB, FK, and DS). The size of the contrast gap between detection and identification for the luminance (closed symbols) and chromatic (open symbols) stimuli. For the chromatic data, circles are for MF-versus-SQ discrimination, squares for  $F$ -versus- $3F$  discrimination. The error bars are 95%-confidence limits obtained through bootstrap analysis (see text for details). The dotted zero line represents detection threshold.

### 5 Experiment 3. Simultaneous detection and identification of $F$ and $3F$

Is the inability to discriminate SQ from MF isoluminous waveforms at detection threshold due to insufficiently narrow chromatic-channel bandwidths? The fundamental and first harmonic component of the chromatic SQ stimulus could be being detected by the same channel, making discrimination of chromatic SQ and MF stimuli impossible at detection threshold. To test this possibility, two subjects, SB and FK, performed a simultaneous-detection-and-identification experiment, in which the two stimuli were the fundamental,  $F$ , and the third harmonic,  $3F$ . These sine waveforms corresponded to the SQ and MF stimuli used in the previous experiment, with all except their lowest harmonics removed. The results are shown in figure 6 as the open-symbol, dashed-line, plots. As can be seen, for both subjects the plots lie very close to detection threshold and below the SQ-versus-MF data, though for subject FK the difference between the two sets of results is quite small. The ratios of identification to detection thresholds for the  $F$ -versus- $3F$  data were 1.1 for subject SB and 1.2 for subject FK. It would seem therefore that the contrast gap between detection and identification of chromatic SQ and MF waveforms is not entirely attributable to limitations in channel bandwidth.

## 6 Discussion

If a chromatic missing-fundamental stimulus looks identical to a square wave at low contrasts, then we would expect a difference between the thresholds for detecting and identifying the two stimuli, providing contrast is not a cue to identification. When contrast was eliminated as a cue to identification by using a simultaneous-detection-and-identification paradigm, we found an identification/detection ratio of just less than a factor of two for these stimuli. While this was substantially lower than that for the luminance stimuli (ratios of 7.1 to 4.0, depending on spatial frequency), it is nevertheless significant. We subsequently showed that  $F$  and  $3F$  chromatic gratings were discriminable very close to detection threshold, suggesting that the spatial bandwidths of the chromatic channels involved were sufficiently narrow to support, in principle, the separate identification of chromatic missing fundamentals and square waves at detection threshold. Moreover the  $F$ -versus- $3F$  results suggested that there was nothing intrinsically difficult about the simultaneous-detection-and-identification task.

Could chromatic aberration account for these results? The chromatic stimuli contained sharp edges, along which narrow luminance contours could arise owing to the slight overlap in the retinal images of the red and green regions. The subjects' task, however, was to identify the chromatic profile of the stimulus: either a gradual transition across the central bar in the case of the missing fundamental, or uniformity in the case of the square wave. It is difficult to see how the presence of a thin luminance contour at the edges of the central bar would hinder this discrimination, and for this reason chromatic aberration is an unlikely cause of the effects measured here.

Do these results suggest that Campbell et al's (1978) model of the missing fundamental is incorrect for isoluminous stimuli? According to the model, the missing-fundamental illusion breaks down once the fundamental becomes independently detectable. The data from experiment 1 show that our chromatic square waves are detected via their fundamental, so the model would predict that chromatic SQ and MF stimuli should be discriminable at, or very close to threshold. The departure from this prediction, while significant, is not, however, very large (less than a factor of two) and it would therefore be imprudent to reject Campbell et al's model on the basis of these data.

One possible explanation for the small difference between the thresholds for detection and identification of chromatic square waves and missing fundamentals is that it reflects the operation of mechanisms involved in the related chromatic Craik–Cornsweet illusion (van den Brink and Kleemink 1976; Ware and Cowan 1983; Neri and Growney 1989; Cole et al 1992; Wachtler and Wehrhahn 1997). The chromatic Craik–Cornsweet illusion refers to induced colour differences on either side of a highpass-filtered (or equivalent) chromatic step-edge, and is conventionally measured by a matching or cancellation technique. It is typically measured at contrasts above the threshold for discriminating the Craik–Cornsweet edge from its step-edge parent, and may thus be considered a 'suprathreshold' version of the missing fundamental. As with the missing fundamental, the chromatic Craik–Cornsweet illusion is much weaker than its luminance counterpart. For example, in one study, Cole et al (1992) found the maximum effect to be about a quarter of its achromatic counterpart. On the plausible assumption that the mechanisms for the missing-fundamental and Craik–Cornsweet illusions are related, it is perhaps not surprising that there is a small region of detectable contrasts at which the chromatic missing fundamental looks indiscriminable from its parent square wave. The sharp edges provided by the higher harmonics in both square wave and missing fundamental alike are presumably implicated in the perceived uniformity of the bars in both stimuli at very low contrasts. This could be instantiated either by some kind of 'filling in' mechanism between the edges (Grossberg and Todorović 1988; Paradiso and Nakayama 1991), or via a symbolic interpretation of the neural image (Watt and Morgan 1985; Kingdom and Moulden 1992).

In conclusion, we have demonstrated the presence of a weak missing-fundamental illusion at isoluminance, one not expected given the shape of the chromatic-contrast-sensitivity function and the characteristics of chromatic spatial channels. It is likely that the illusion involves mechanisms similar to those responsible for the chromatic Craik–Cornsweet illusion.

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